Equation of state for an ideal gas from Heisenberg's uncertainty principle

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Abstract

Equation of state for an ideal gas it is the relation used to study of ideal gases on the macroscopic scale. Here, we are concerned with the relationships among such quantities as pressure, volume,and temperature of a gas.but the Heisenberg's uncertainty principle used in the quantum mechanics in the microstate, in this paper we will try to connect between the microstate and macrostate by derive Equation of state for an ideal gas from Heisenberg's uncertainty principl.

1 Introduction

The ideal gas law is an expression that relates the volume, pressure, and temperature of a gas. This relationship can be written as follows

$$PV = Nk_BT \tag{1}$$

Which P is pressure, V volume ,N number of gas particles,k_B Boltzmann's constant, T temperature .It was first stated by Benoît Paul Clapeyron in 1834 as a combination of the empirical Boyle's law, Charles's law, Avogadro's law, and Gay-Lussac's law. the parameters in the ideal gas law are macroscopic parameters but in the Heisenberg's uncertainty principle it used to study the microscopic parameters , the different between macroscopic parameters and microscopic parameters under the Heisenberg's uncertainty principle It tells us that there is a fuzziness in nature, a fundamental limit to what we can know about the behaviour of quantum particles and, therefore, the smallest scales of nature. Of these scales, the most we can hope for is to calculate probabilities for where things are and how they will behave.but how to move from prediction by everything to the uncertainty principle enshrines a level of fuzziness into quantum theory.

2 Method

let some system has N number of gas particles and temperature T which it equal the sum of n Planck temperature T_P , then the total average uncertainties of all matter we can write in uncertainty principle as

$$\Delta p \, \Delta X \; \geq \; nN\hbar \tag{2}$$

We will partly repeat that derivation here, but we also develop some important new insights. Heisenberg's uncertainty principle is given by

$$\Delta p \, \Delta X \; \sim \; nN\hbar \tag{3}$$

since multiple the left hand side by one over the uncertainty in the time and the uncertainty in the time$\frac{\Delta t}{\Delta t}$,so we get

$$\Delta p \, \frac{\Delta t}{\Delta t} \Delta X \; \sim \; nN\hbar \tag{4}$$

from the Newton's second law$\Delta F \; = \frac{\Delta p}{\Delta t}$

$$\Delta F \;\; \Delta t \, \Delta X \; \sim \; nN\hbar \tag{5}$$

since multiple the left hand side by one over the square of uncertainty in the position and the square of uncertainty in the position $\frac{(\Delta X)^2}{(\Delta X)^2}$,so we get

$$\Delta F \;\; \Delta t \, \frac{(\Delta X)^3}{(\Delta X)^2} \sim \; nN\hbar \tag{6}$$

using the definition of the pressure ΔP=$\frac{\Delta F}{\Delta A}$,which ΔA the uncertainty in the Area ,In microscopic scale we can show that $\Delta A = (\Delta X)^2$,so we get

$$\Delta P \;\; \Delta t \; (\Delta X)^3 \sim \; nN\hbar \tag{7}$$

In microscopic scale we can show that $\Delta V = (\Delta X)^3$,which ΔV the uncertainty in the Volume, so we get

$$\Delta P \;\; \Delta t \, \Delta V \; \sim \; nN\hbar \tag{8}$$

temperature T which it equal the sum of n Planck temperature T_P,[2] so we get

$$\Delta P \;\; \Delta V \; \sim \; \frac{T \; N \; \hbar}{T_P \; \Delta t} \tag{9}$$

The Planck temperature is the unit of temperature in the system of Planck units. It has the value

$$T_P = \frac{m_P c^2}{k_B} \tag{10}$$

Which m_P is Planck mass[2] ,so we get

$$\Delta P \ \ \Delta V \ \sim \ \frac{\hbar}{m_P c^2 \ \Delta t} k_B N T \tag{11}$$

Now we assume that the uncertainty of the average time (Δt) is the planck time t_P then the uncertainty of the average pressure (ΔP) will be P and the uncertainty of the average volume (ΔV) will be V .

$$P \ \ V \ \sim \ \frac{\hbar}{m_P c^2 \ t_P} k_B N T \tag{12}$$

The Planck time is the unit of time in the system of Planck units. It has the value

$$t_P = \frac{m_P c^2}{\hbar} \tag{13}$$

so we get

$$P \ \ V \ \sim k_B N T \tag{14}$$

this is The ideal gas law

3 Conclusion

From this derivation we can study the macroscopic scale by the uncertainty principle , using Planck units and the uncertainty principle we can move from the quantum state to the macroscopic state becouse Planck units are only one of several systems of natural units, but Planck units are not based on properties of any prototype object or particle (the choice of which is inherently arbitrary), but rather on only the properties of free space. They are relevant in research on unified theories such as quantum gravity. perhaps this will be important way to unification of physics .

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5 References

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